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NONSTATIONARY HYDRODYNAMIC CHARACTERISTICS OF A DOUBLE GRID OF PROFILES

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The flow about a double grid of solid profiles of arbitrary shape which vibrate in a stream of an ideal incompressible fluid is considered. Behind the grid profiles, the nonstationary vortex traces simulated by the lines of contact velocity discontinuity are taken into account. The problem is reduced to the solution of a system of two integral equations relative to the fluid velocity on the initial profiles of the double grid under the assumption that the vibration amplitudes are small. Formulas for calculating the nonstationary forces and moments are derived. The dependences of these forces on the shape, mutual positions, and laws of vibration of the grid profiles are studied.

Introduction. The blade rims of turbomachines are often designed from double (main and auxiliary) blades (Fig. 1). In particular, these rims are used in mine ventilators operating at varied air supply owing to the change in the angles at which the blades are installed by special rotary devices. The rotation of the double blades of the ventilator's rotary wheels is preferred from the engineering viewpoint owing to the decrease in the number of expensive rotary devices and the choice of the axis of rotation which ensures zero centrifugal moments of inertia [1]. The problem of mutual arrangement of the main and auxiliary blades with a view to optimizing their hydrodynamic characteristics has not yet been studied adequately because of the absence of effective and reliable programs of calculation of these characteristics. In this paper, an algorithm of calculating the stationary and nonstationary forces and the moments of the profiles of a double grid which vibrate in a flow of an ideal incompressible fluid is constructed. For a single grid, the algorithm was constructed in [2], and it is reduced to the solution of one integral equation on the grid profile. One managed to generalize this algorithm in the case of a double grid, but the problem is reduced to the solution of a system of two integral equations on the main and auxiliary profiles. The algorithm is realized as a computer program.

The dependence of the stationary and nonstationary forces and the moments which occur on the main and auxiliary profiles of a double grid on their shape, the mutual position, and the laws of vibrations is studied. It is shown, in particular, that the phase shift between the vibrations of the main and auxiliary profiles is one of the key parameters which affect considerably the level of nonstationary forces.

Formulation of the Problem. We shall consider the potential nonstalling flow of an ideal incompressible fluid about a double grid of vibrating profiles in the plane of the complex variable $z = x + iy$. The fluid velocity $\overline{V}_{-\infty}$ is assumed to be constant at an infinite distance before the grid. The profiles of the main and auxiliary grids are considered arbitrary and the output edges of the profiles are regarded as the points of reversal. We assume that the profiles vibrate with small amplitudes A according to the following harmonic law of the equal frequency ω with a constant phase shift μ between the adjacent profiles:

$$z_n^k(s,t) = z_0^k(s) + inh + Ag^k(s)\exp\left(j(\omega t + n\mu)\right).$$

Here the subscript $k = 1$ and 2 denotes the main and auxiliary profiles, respectively, $z_0^k(s)$ is the complex coordinate of the point s of the contour in the middle (nondeformed) position, $g^1 = g_x^1 + ig_y^1$ and $g^2 =$

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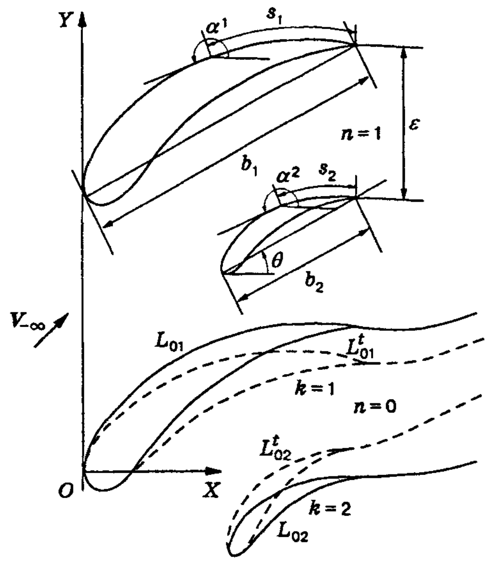

Fig. 1

$(g_x^2 + ig_y^2)\exp(j\mu_{12})$ are the vibration modes of the main and auxiliary profiles, μ_{12} is the phase shift between them, h is the grid step, n is the double-profile number, and $\mu = 2\pi m/N$ $(m = \overline{1, N-1})$. Hereinafter, the imaginary unit j is introduced to describe time processes and it does not interact with the imaginary quantity i $(i \cdot j \neq -1)$.

Behind the profiles, we take into account the nonstationary vortex traces (leaving the target edges because of the change in the circulation on the profiles), which is simulated by the contact-discontinuity lines of velocity.

By virtue of the potential character of the flow, the complex velocity of the fluid $\overline{V}_\Sigma(z,t)$ is an analytic function of z everywhere outside the vibrating profiles of the double grid and the vortex traces behind them. We present it in the form

$$\overline{V}_\Sigma(z,t) = \overline{V}_0(z) + \overline{V}(z)\exp(j\omega t),$$

where $\overline{V}_0(z)$ is the flow velocity for a fixed double grid and $\overline{V}(z)$ is the nonstationary velocity perturbation caused by small perturbations.

The function $\overline{V}_\Sigma(z,t)$ should satisfy the following conditions:

(1) $\overline{V}_0(-\infty) = \overline{V}_{-\infty}$ and $V(-\infty) = 0$ at infinity before the grid;

(2) Periodicity of $\overline{V}_0(z + inh) = \overline{V}_0(z)$ and $\overline{V}(z + inh) = \overline{V}(z)\exp(jn\mu)$;

(3) No-flow at the boundaries of the profiles $\overline{V}^k_{\Sigma rn}(\zeta, t) = V^k(s,t)\exp(-i\alpha^k_n(s,t))$, where $V^k(s,t) = V^k_0(s) + V^k(s)\exp(j(\omega t + n\mu))$ is the modulus of the relative fluid velocity, $\overline{V}^k_{\Sigma rn} = \overline{V}^k_{\Sigma n} - j\omega A\bar{g}^k(s)\exp(j(\omega t + n\mu))$ is the relative fluid velocity on the nth profile of the grid, $\alpha^k_n(s,t) = \theta^k(s) + \alpha^k(s)\exp(j(\omega t + n\mu))$ is the angle between the tangent to the vibrating contour L^t_{nk} and the OX axis, $\theta^k(s)$ is the angle between the tangent to the fixed contour L_{nk} and the OX axis, and s is the length of the contour arc L_{nk} reckoned from the output edge in the direction of the positive pass around the contour;

(4) Pressure continuity (the dynamic condition) and the normal velocity component of the fluid (the kinematic condition) upon passage through the vortex traces. In view of the low vibration frequency, we assume [2] that the vortex traces are located along the streamlines of the steady-state flow which leaves the output edges of the profiles;

(5) Joukowski–Chaplygin condition at the acute output edges, which has the form

$$V^k_0(+0) + V^k_0(-0) = 0, \qquad V^k(+0) + V^k(-0) = -\frac{j\omega}{V^k_0(0)} \int\limits_{L^t_{0k}} V^k(\sigma)\,d\sigma$$

with allowance for the dynamic condition in the traces (the condition for the velocity jump at the acute output edge).

Method of Solution. According to the generalized Cauchy formula, with allowance for the condition at infinity before the grid we have

$$\overline{V}_\Sigma(z,t) = \frac{1}{2iNh}\sum_{k=1}^{2}\sum_{n=0}^{N-1}\int_{L_{0k}^t} \overline{V}_\Sigma(\zeta,t)\Big\{\coth\Big[\frac{\pi}{Nh}(z-\zeta^k-inh)\Big]+1\Big\}d\zeta^k$$

$$+\frac{1}{2iNh}\sum_{k=1}^{2}\sum_{n=0}^{N-1}\int_0^\infty \eta_n^k(\tau,t)\Big\{\coth\Big[\frac{\pi}{Nh}(z-\zeta^{*k}(\tau)-inh)\Big]+1\Big\}d\tau^k+\overline{V}_{-\infty},$$

where the tangent discontinuity of the velocity on the line of vortex trace is

$$\eta_n^k = -\frac{1}{V_0^k(\tau)}\frac{\partial}{\partial t}\Big\{\mathrm{Re}_i\int_{L_{nk}^t} V_{\Sigma n}(\zeta^k,t)\,d\zeta^k\Big\}\Big|_{t=t^k},\qquad t^k = t-\int_0^\tau \frac{d\tau}{V_0^k(\tau^k)}.$$

We perform a limiting passage from the domain of analyticity $\overline{V}_\Sigma$ to the profiles of a double grid by means of the Sokhotskii–Plemelj formulas. By virtue of the small vibrations, we linearize the stationary solution V_0. Here the boundary conditions are displaced to the stationary positions of the profiles, and integration over the moving contours L_{0k}^t is replaced by integration over the fixed contours L_{0k}, ignoring second-order quantities compared to the amplitude A of vibrations. We write the terms with the factor $\exp(j\omega t)$ in the resulting equalities. Taking into account the Joukowski–Chaplygin condition of unambiguous resolvability and using the actual part with respect to i, we obtain a system of integral equations relative to the values of the stationary velocities $V_0^k(s)$ and the amplitude values of the nonstationary velocities $V^k(s)$ on the grid profiles, which has the unique solution

$$V_0^p(s)-\sum_{k=1}^{2}\int_{L_0^k} V_0^k(\sigma)\Phi_{10}^{pk}(s,\sigma)\,d\sigma^k = \Pi_{10}^p(s),$$

$$V^p(s)-\sum_{k=1}^{2}\int_{L_0^k} V^k(\sigma)\Phi_1^{pk}(\mu,s,\sigma)\,d\sigma^k = A\cdot\Pi_1^p(s),\quad p=1,\,2, \tag{1}$$

where

$$\Phi_{10}^{pk}(s,\sigma)=\Phi_0^{pk}(s,\sigma)-0.5(\Phi_0^{pk}(+0,\sigma)+\Phi_0^{pk}(-0,\sigma)),$$

$$\Phi_0^{pk}(s,\sigma)=\mathrm{Re}_i\Big\{\frac{\exp(i\theta^p(s))}{ih}\Big(\coth\Big[\frac{\pi}{h}(z_0^p-\zeta_0^k)\Big]+1\Big)\Big\},$$

$$\Pi_{10}^p(s)=\Pi_0^p(s)-0.5(\Pi_0^p(+0)+\Pi_0^p(-0)),\qquad \Pi_0^p(s)=\mathrm{Re}_i\{2\exp(i\theta^p(s))\overline{V}_{-\infty}\},$$

$$\Phi_1^{pk}(\mu,s,\sigma)=\Phi^{pk}(\mu,s,\sigma)-0.5(\Phi^{pk}(\mu,+0,\sigma)+\Phi^{pk}(\mu,-0,\sigma)),$$

$$\Phi^{pk}(\mu,s,\sigma)=\mathrm{Re}_i\Big\{-\frac{\exp(i\theta^p(s))}{iNh}\sum_{n=0}^{N-1}\exp(jn\mu)\Big\{\coth\Big[\frac{\pi}{Nh}(z_0^p-\zeta_0^k-inh)\Big]+1$$

$$-j\omega\int_0^\infty\frac{\exp(-j\omega T^k(\tau))}{V_0^k(\tau)}\Big\{\coth\Big[\frac{\pi}{Nh}(z_0^p-\zeta_0^{*k}-inh)\Big]+1\Big\}d\tau\Big\}\Big\},$$

$$\Pi_1^p(\mu,s,\sigma)=\Pi^p(\mu,s,\sigma)-0.5(\Pi^p(\mu,+0,\sigma)+\Pi^p(\mu,-0,\sigma)),$$

$$\Pi^p(\mu,s,\sigma)=\sum_{k=1}^{2}\int_{L_0^k} V_0^k(\sigma)D^{pk}(\mu,s,\sigma)\,d\sigma^k+j\omega G^p(\mu,s),$$

$$D^{pk}(\mu,s,\sigma)=\mathrm{Re}_i\Big\{-\frac{\pi\exp(i\theta^p(s))}{iN^2h^2}\sum_{n=0}^{N-1}(g(s)-g(\sigma)\exp(jn\mu))\sinh^{-2}\Big[\frac{\pi}{Nh}(z_0^p-\zeta_0^k-inh)\Big]\Big\},$$

$$G^p(\mu,s)=\mathrm{Re}_i\Big\{2\exp(i\theta^p(s))\Big[-\bar g^p+\sum_{k=1}^{2}\int_{L_0^k}\bar g^k(\sigma)\Phi^{pk}(\mu,s,\sigma)\,d\zeta_0^k\Big]\Big\}.$$

The system of integral equations (1) was solved numerically. The profile was replaced by a polygon inscribed in it [3], and the integral over the trace was calculated from the Filone quadrature formulas for the integrals of varying functions [4]. The resulting system of linear algebraic equations was solved by the Gauss method with the choice of the maximum element.

Calculation of the Forces and the Moment. On the initial ($n=0$) profile, the formulas to calculate the forces

$$X^p=0.5\rho\overline{V}_{-\infty}^2h(C_{0X}^p+C_X^pA\exp(j\omega t)),\qquad Y^p=0.5\rho\overline{V}_{-\infty}^2h(C_{0Y}^p+C_Y^pA\exp(j\omega t))$$

and the moment relative to the input edges of the profile

$$M^p=0.5\rho\overline{V}_{-\infty}^2h(C_{0M}^p+C_M^pA\exp(j\omega t))$$

where ρ is the density of the fluid, and C_{0X}^p, C_{0Y}^p, C_{0M}^p, C_X^p, C_Y^p, and C_M^p are the corresponding dimensionless factors, are derived by means of the Cauchy–Lagrange integral and have the form [1, 5]

$$C_{0X}^p-iC_{0Y}^p=i\int_{L_0^p}(V_0^p(s))^2\exp(-i\alpha^p(s))\,ds^p,\qquad C_{0M}^p=\mathrm{Re}\int_{L_0^p}(V_0^p(s))^2\bar z^p\,d\bar z^p,$$

$$C_X^p-iC_Y^p=2i\int_{L_0^p}\{V_0^p(s)V^p(s)+j\omega V_0^p(s)\mathrm{Re}(g^p(s)\exp(-i\alpha^p(s)))\}\,ds^p$$

$$-i\int_{L_0^p}(V_0^p)^2(s)\frac{dg^p}{dz^p}\exp(-i\alpha^p(s))\,ds^p-2ij\omega\int_{L_0^p}\{V^p(s)+j\omega\mathrm{Re}(g^p(s)\exp(-i\alpha^p(s)))\}(\overline{z^p-z_*^p})\,ds^p,$$

$$C_M^p=-2\int_{L_0^p}\{V_0^p(s)V^p(s)+j\omega V_0^p(s)\mathrm{Re}(g^p(s)\exp(-i\alpha^p(s)))\}\mathrm{Re}(z^pd\bar z^p)\,ds^p$$

$$+\int_{L_0^p}(V_0^p(s))^2\mathrm{Re}\Big(\frac{dg^p}{dz^p}z^p\,dz^p\Big)-\int_{L_0^p}(V_0^p(s))^2\mathrm{Re}(g^p\,d\bar z)$$

$$+j\omega\int_{L_0^p}\{V^p(s)+j\omega\mathrm{Re}(g^p(s)\exp(-i\alpha^p(s)))\}(|z^p|^2-|z_*^p|^2)\,ds^p.$$

Here z_*^p is the complex coordinate of the acute output edge of the profile L_0^p. These formulas are convenient to use because they contain only the solutions of the integral equations (1) $V^p(s)$ and $V_0^p(s)$.

Results. Our algorithm is realized as a computer program for calculating both stationary and nonstationary characteristics. Generally, to obtain three correct signs after a comma, one needs to specify no more than 30 points on the main and auxiliary profiles of a double grid and no more than 100 points in the traces behind them. The computing time of one variant on a AT-486 computer does not exceed 3 min. To check the work of the program, we made a comparison with [5] for the case where the auxiliary profile coincides with the main profile and was located under the latter at a distance of the half-step of a double grid. The vibration modes were set identical as well: $g^1(s)=g^2(s)$. In this case, the double grid became equivalent to a single grid with the halved step. For all the variants of calculation, the stationary and nonstationary velocities and forces almost completely coincide (the discrepancy is no more than 0.1 %).

To study the behavior of the stationary and nonstationary hydrodynamic characteristics, a double grid

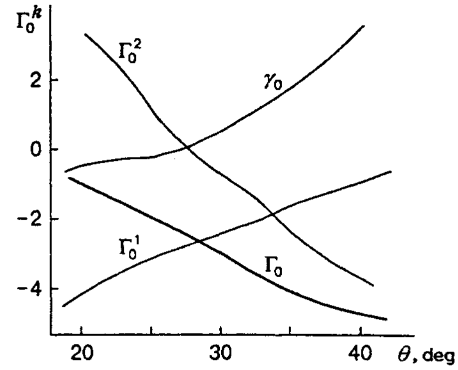

Fig. 2

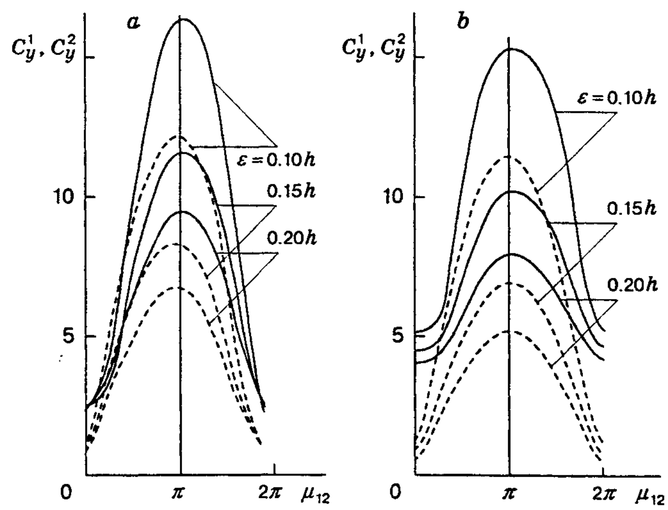

Fig. 3

in flow with $\bar{V}_{-\infty} = \exp(-i\pi/4)$ and the initial profiles shown in Fig. 1 was taken. We shall characterize the position of the auxiliary profile relative to the basic parameters ε (the distance along the front between the output edges of the profiles) and θ (the angle of installation of the auxiliary profile relative to the input edge; $\theta = 0$ corresponds to the initial position of the auxiliary profile shown in Fig. 1).

Figure 2 shows the stationary circulation Γ_0^k which characterizes the rotation of the flow of the main ($k = 1$) and auxiliary ($k = 2$) grids versus θ. Obviously, the quantities $\Gamma_0 = \Gamma_0^1 + \Gamma_0^2$ and $\gamma_0 = \Gamma_0^2/\Gamma_0^1$ characterize the total rotation of the flow by the double grid and the relative contribution from each grid to this rotation, respectively. One can see on the diagrams that Γ_0 and γ_0 depend strongly on the position and the angle of installation of the auxiliary grid relative to the main grid, especially at small distances between them.

Figure 3 shows the dependences of the amplitude of the circumferential components of the nonstationary forces C_y^1 (solid curves) and C_y^2 (dashed curves) during bending [Fig. 3a, $g^1 \equiv i$ and $g^2 \equiv i \cdot \exp(j\mu_{12})$] and torsional [Fig. 3b, $g^1 = i(z_0^1(s) - z_{**}^1)$ and $g^2 = i(z_0^2(s) - z_{**}^2)\exp(j\mu_{12})$, where z_{**}^1 and z_{**}^2 are the coordinates of the input edges of the initial profiles] vibrations on the phase shift μ_{12} for different positions of the auxiliary profile relative to the main profile ($\varepsilon/h = 0.10$, 0.15, and 0.20, $\theta = 30°$, and the Strouhal number $\mathrm{Sh} = \omega h/V_{-\infty} = 1$). One can see that the maximum of the amplitude is observed when the main and auxiliary profiles change in antiphase, and this maximum increases as the distance ε between them becomes smaller [the behavior of the curves C_x^p and C_M^p ($p = 1$ and 2) is qualitatively similar]. This is connected with the fact that the width of the blade-to-blade channel between the main and auxiliary profiles changes during vibrations with the phase shift μ_{12}. Evidently, for a fixed ε, the change is maximal for

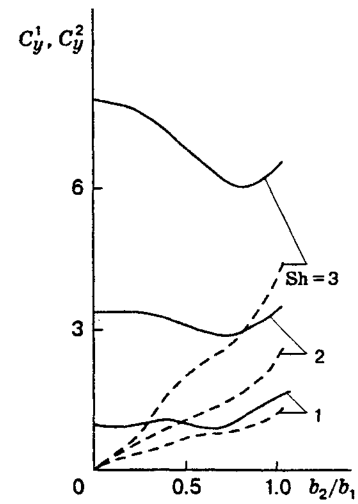

Fig. 4

antiphase vibrations ($\mu_{12} = \pi$); the smaller ε, the greater the relative change in the channel width.

Figure 4 shows the dependences of the amplitude of the circumferential component of the nonstationary forces C_y^1 (solid curves) and C_y^2 (dashed curves) on the relative dimension of the auxiliary profile (the chords b_2) for various frequencies of flexural vibrations ω ($g^1 \equiv i$, $g^2 \equiv i$, $\mu_{12} = 0$, $\varepsilon = 0.3h$, and $\theta = 30°$). In the calculations, the chord b_2 of the auxiliary profile was changed, while the profile itself remained unchanged. As one should expect, the level of nonstationarity (the moduli of the forces C_y^p, where $p = 1$ and 2) increases with increase in ω (Sh $= \omega h/V_{-\infty}$ increases). Here the dependence of the amplitudes of the nonstationary forces on the profile dimension has a complicated character and, in particular, can be nonmonotone.

The examples show that the mutual arrangement of the profiles of a double grid, their relative dimensions, and the phase shift between their vibrations affect greatly the stationary and nonstationary hydrodynamic characteristics of the double grid. This circumstance should be taken into account in the choice of the optimum geometrical parameters of double blades.

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